SOME PROBLEMS IN THE THEORY OF ELASTICITY FOR A SEMI-INFINITE STRIP

(NEKOTORYE ZADACHI TEORII UPRUGOSTI DLIA POLUPOLOSY)

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The authors consider the problem of a symmetrically loaded semi-infinite strip clamped along the short edge. An integral equation is derived for the normal stress at the clamp and the nature of the singularities of its solution is investigated. A detailed study is made of the case when normal and tangential forces act on a longitudinal edge. For such loading the nature of the stress distribution at the clamp is investigated in detail. Numerical calculations are given for the case when the strip is compressed by two point forces.

In many works dealing with semi-infinite strips, wide use is made of methods based on the principle of localization. In this way a satisfactory solution is obtained a long way from singular points such as: 1. points at which the nature of the boundary conditions changes; 2. points at which the loading becomes discontinuous; 3. corner points. By these methods the singularities of the solution which occur in the aforementioned cases and reduce its accuracy, are excluded from it.

Horvay [1 to 3] has studied the problem of a semi-infinite strip with free longitudinal edges and with self-equilibrated loads acting at the end. He selects a stress function in the form

$$\varphi(x, y) = \sum C_k f_k(y)\, g_k(x)$$

where a complete system of orthogonal polynomials is used for $f_k(y)$ and the multipliers $g_k(x)$ are determined from the condition of minimum potential energy.

Koiter and Alblas [4] have studied the problem of the extension of a semi-infinite strip with free longitudinal edges and with a clamped short edge. By applying a Fourier sine transformation to the equation for the stress function they, as all subsequent authors, reduced the solution of the problem approximately to an infinite system of linear algebraic equations.

In papers by Pickett and Jyengar ([5 and 6] 1956, 1962) the stress functions are sought in the form of combinations of Fourier series and integrals.

In 1958 Zorski [7] solved the problem for a plate in the form of a semi-infinite strip. By seeking a solution in the form of combinations of solutions of two potentials he reduced the problem to a singular integral equation.

In a paper by Theocaris ([8], 1959) dealing with the problem of a semi-infinite strip with free longitudinal edges and with a concentrated force acting on the end, an energy method was used.

Teodorescu ([9], 1960) suggested the method resembling that of Pickett and Jyengar for the solution of the problem of a semi-infinite strip.

A variational method was used by Trapeznikov ([10], 1963) for the solution of the problem of a semi-infinite strip compressed by two concentrated forces; the short side of the strip was stress free.

Gaydon and Shepherd ([11], 1964) studied Horvay's problem by means of the method of homogeneous solutions. Since the homogeneous solutions were not orthogonal, in order to satisfy the conditions at the end, they were expanded into series of orthogonal functions.

Homogeneous solutions have also been used by Buchwald ([12], 1964).

All of the aforementioned authors ignored the singularities of the solution and for this reason the linear systems to which the problem was reduced were often inexact.

The paper by Benthem ([13], 1963), which solves the problem of a semi-infinite strip as stated by Koiter and Alblas, is free of this shortcoming. By applying Laplace transforms to the equation for the stress function Benthem reduced the problem to a system of linear algebraic equations. He established the nature of the singularity of the solution on the basis of corresponding problems for a quarter plane studied earlier by Williams ([14 and 15], 1952 and 1956). Computation of the singularity given in the present paper assumes a form different from that of Benthem [13], with resulting improvement in the accuracy of the solution. It should also be noted that the problem studied here has not been studied in the works listed above.

1. Consider the symmetrical problem for a semi-infinite strip with the following boundary conditions (Fig. 1):

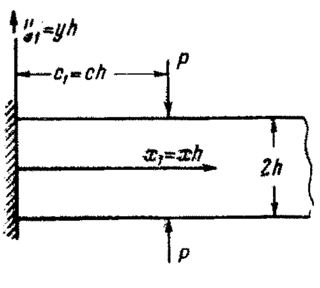

FIG. 1.

$$u = v = 0, \qquad x = 0 \tag{1.1}$$

$$\sigma_{y_1} = g\,(x), \quad \tau_{x_1 y_1} = r\,(x)\;\mathrm{sgn}\; y, \quad y = \pm 1 \tag{1.2}$$

Here u and v are the displacements along the axes of x_1 and y_1 respectively; $\tau_{x_1 y_1}$, and σ_{y_1} are the tangential and normal stresses.

In order to derive the integral equation for the problem we make use of the fundamental relations of the plane theory of elasticity in terms of displacements

$$\nu \frac{\partial\theta}{\partial x} + \triangle u = 0, \qquad \frac{\sigma_{x_1} h}{G} = (\nu - 1)\theta + 2\frac{\partial u}{\partial x}, \qquad \frac{\tau_{x_1 y_1} h}{G} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{1.3}$$

$$\nu \frac{\partial\theta}{\partial y} + \triangle v = 0, \qquad \frac{\sigma_{y_1} h}{G} = (\nu - 1)\theta + 2\frac{\partial v}{\partial y}, \qquad \gamma = \frac{1}{1-2\sigma}$$

Here σ is Poisson's ratio and G is the shear modulus. Let the boundary conditions be given by (1.1) and (1.2) and let us represent u and v as the sum of the solutions of two auxiliary problems:

(1) The problem for a semi-infinite plane with a boundary along the y-axis and with the boundary conditions

$$x = 0, \qquad v(y) = 0, \qquad \sigma_{x_1}(y) = \sigma(y), \qquad |y| < \infty \tag{1.4}$$

(2) The problem for a strip with boundary conditions

$$x = 0, \qquad v(y) = 0, \qquad \sigma_{x_1}(y) = 0 \qquad |y| \leqslant 1 \tag{1.5}$$

$$y = \pm 1, \qquad \sigma_{y_1} = m(x), \qquad \tau_{x_1 y_1} = \pm n(x) \tag{1.6}$$

Here m and n are such that for the general problem boundary conditions (1.2) hold. It is easily seen that the solution of (1) is given by the formulas

$$u = -\left(\frac{2}{\pi}\right)^2 \frac{h}{G} \int_0^\infty \cos\lambda x \, d\lambda \int_0^\infty \frac{Q(s)(\omega^2 + \nu s^2)}{(1+\nu)\omega^4} \cos sy \, ds + C_1 \quad (\omega^2 = \lambda^2 + s^2)$$

$$v = -\left(\frac{2}{\pi}\right)^2 \frac{h}{G} \int_0^\infty \sin\lambda x \, d\lambda \int_0^\infty \frac{Q(s)\,\nu\lambda s \sin sy \, ds}{(1+\nu)\omega^4} \quad \left(Q(s) = \int_0^\infty \sigma(t)\cos st \, dt\right) \tag{1.7}$$

Here C_1 is an arbitrary constant.

Solution of (2) is obtained by applying a Fourier sine transformation to the system of equations (1.3). Without describing the intermediate computations, we state the final formulas

$$u = \frac{2}{\pi} \int_0^\infty (A \cosh\lambda y + B\lambda y \sinh \lambda y) \cos\lambda x \, d\lambda + C_2$$

$$v = \frac{2}{\pi} \int_0^\infty \left[A \sinh\lambda y + B\left(\lambda y \cosh\lambda y - \frac{2+\nu}{\nu} \sinh\lambda y\right)\right] \sin\lambda x \, d\lambda \tag{1.8}$$

Here $A(\lambda)$ and $B(\lambda)$ are arbitrary functions; C_2 is an arbitrary constant. From formulas (1.7) and (1.8) we can easily see that for the problem obtained by superimposing solutions of (1) and (2) the following boundary conditions hold:

$$x = 0, \qquad v(y) = 0, \qquad \sigma_{x_1} = \sigma(y), \qquad |y| \leqslant 1 \tag{1.9}$$

$$y = 1, \qquad \sigma_{y_1} = N(x) + m(x), \qquad \tau_{x_1 y_1} = M(x) + n(x), \quad x > 0 \tag{1.10}$$

Here $N(x)$ and $M(x)$ are, respectively, the normal and tangential stresses in the semi-infinite plane at $y = 1$. In order to satisfy conditions (1.2) we put

$$m(x) = g(x) - N(x), \qquad n(x) = r(x) - M(x) \tag{1.11}$$

Then

$$A(\lambda) = \frac{g_1 - N_1}{\lambda\Delta_+}\left(\lambda \cosh \lambda - \frac{\sinh \lambda}{\nu}\right) + \frac{M_1 - r_1}{\lambda\Delta_+}\left(\lambda \sinh \lambda - \frac{\nu + 1}{\nu} \cosh \lambda\right)$$

$$B(\lambda) = -\frac{M_1 - r_1}{\lambda\Delta_+}\cosh \lambda - \frac{g_1 - N_1}{\lambda\Delta_+} \sinh \lambda,$$

$$g_1 = \frac{h}{G}\int_0^\infty g(x)\sin \lambda x\, dx, \quad \Delta_+ = \sinh 2\lambda + 2\lambda, \quad r_1 = \frac{h}{G}\int_0^\infty r(x)\cos \lambda x\, dx$$

$$M_1 = \frac{h}{G(2 + 2\nu)}\int_0^\infty \sigma(t)\, e^{-\lambda\beta}(\nu + 1 - \nu\lambda\beta)\, dt$$

$$N_1 = \frac{h}{G(2 + 2\nu)}\int_0^\infty \sigma(t)\, e^{-\lambda\beta}(\nu\lambda\beta - 1)\, dt \qquad (\beta = |1 - t|)$$

The solution obtained by superimposing (1) and (2) satisfies all the boundary conditions except $u = 0$ at $x = x_1 / h = 0$. Instead of this we obtain

$$u|_{x=0} = \frac{\nu + 2}{2\pi(1 + \nu)}\frac{h}{G}\int_{-\infty}^{\infty} \sigma(t)\ln|t - y|\, dt + \frac{h}{G\pi(1 + \nu)}\int_{-\infty}^{\infty} \sigma(t)K(t, y)\, dt +$$

$$+ \frac{2}{\pi}\int_0^\infty \frac{g_1(\lambda)}{\lambda\Delta_+}\left[\lambda(\cosh\lambda\cosh\lambda y - y\sinh\lambda\sinh\lambda y) - \frac{\sinh\lambda\cosh\lambda y}{\nu}\right]d\lambda - \tag{1.12}$$

$$- \frac{2}{\pi}\int_0^\infty \frac{r_1(\lambda)}{\lambda\Delta_+}\left[\lambda(\sinh\lambda\cosh\lambda y - y\cosh\lambda\sinh\lambda y) - \frac{\nu + 1}{\nu}\cosh\lambda\cosh\lambda y\right]d\lambda +$$

$$+ C_3\frac{h}{G}\frac{\nu + 2}{2\pi(1 + \nu)}, \qquad |y| \leqslant 1$$

$$K(t, y) = \int_0^\infty \frac{e^{-\lambda\beta}}{\lambda\Delta_+}\Big\{\lambda y \sinh\lambda y\,[\cosh\lambda(1 + \nu - \nu\lambda\beta) + \sinh\lambda(1 - \nu\lambda\beta)] -$$

$$- (\cosh\lambda y - 1)\left[\left(\lambda\cosh\lambda - \frac{\sinh\lambda}{\nu}\right)(1 - \nu\lambda\beta) + (\nu + 1 - \nu\lambda\beta)\times\right.$$

$$\left.\times\left(\lambda\sinh\lambda - \frac{\nu + 1}{\nu}\cosh\lambda\right)\right]\Big\}\, d\lambda \qquad (\beta = |1 - t|)$$

Here C_3 is an arbitrary constant which can be determined from the condition of equilibrium of the semi-infinite strip.

If we impose the condition that $u = 0$ at $x \times 0$ we obtain an integral equation in terms of the stress $\sigma(y)$ at the support. Let us put

$$\sigma(y) = \sigma(y) \quad \text{for } |y| \leqslant 1, \qquad \sigma(y) = 0 \quad \text{for } |y| > 1$$

After simple manipulations of (1.12) we obtain an integral equation of the form

$$\int_{-1}^{1} \sigma(t)\left\{\frac{\nu+2}{4(1+\nu)}\ln|t-y| + \frac{\nu^2+2\nu+2}{4\nu(1+\nu)}\ln[(2-t)^2-y^2] - \right.$$
$$\left. - \frac{\nu}{2(1+\nu)}\left[\frac{(1-t)(1-y)}{(2-y-t)^2} + \frac{(1-t)(1+y)}{(2+y-t)^2}\right]\right\}dt -$$
$$- \int_{-1}^{1} \sigma(t)L(t, y)\,dt + C_3\frac{\nu+2}{4(1+\nu)} = f(y), \qquad |y| \leqslant 1 \tag{1.13}$$

$$L(t, y) = \frac{\nu}{1+\nu}\left(\frac{1}{4+y-t} + \frac{1}{4-y-t}\right) - \frac{\nu+2}{4(1+\nu)}\ln[(4-t)^2-y^2] +$$
$$+ \frac{1}{(1+\nu)}\int_0^\infty \frac{(4\lambda - e^{-2\lambda})}{(e^{4\lambda}+2\lambda e^{+2\lambda}-1)\lambda}\left\{\cosh\lambda t\,(\nu\lambda - 1 - \nu\lambda t\tanh\lambda t)\Big[\lambda y\sinh\lambda y - \right.$$
$$-(\cosh\lambda y - 1)\left(\lambda - \frac{2+\nu}{2\nu} - \frac{e^{-2\lambda}}{2}\right)\Big] - \cosh\lambda t\cosh\lambda e^{-\lambda}[(\cosh\lambda y - 1)\times$$
$$\left.\times(\nu + 1 - \nu\lambda\tanh\lambda) + \nu\lambda y\sinh\lambda y]\right\}d\lambda$$

$$f(y) = -\frac{G}{h}\int_0^\infty \frac{g_1(\lambda)}{\lambda\Delta_+}\left[\lambda(\cosh\lambda\cosh\lambda y - y\sinh\lambda\sinh\lambda y) - \frac{1}{\nu}\sinh\lambda\cosh\lambda y\right]d\lambda +$$
$$+ \frac{G}{h}\int_0^\infty \frac{r_1(\lambda)}{\lambda\Delta_+}\left[\lambda(\sinh\lambda\cosh\lambda y - y\cosh\lambda\sinh\lambda y) - \frac{\nu+1}{\nu}\cosh\lambda\cosh\lambda y\right]d\lambda$$

This way we have reduced the problem to an integral equation of the first kind. The kernel of this equation has a removable singularity on the diagonal $y = t$ and a fixed singularity at the points $y = \pm 1$. The fixed singularity in the kernel complicates the investigation of the equation as well as its numerical solution.

2. Let us now investigate the singularity of the solution of the problem under consideration. It can easily be assumed that the nature of this singularity will be the same as that at the corner in the solution of the analogous problem for a quarter plane. We shall use the results of Williams [14 and 15], Ufliand [16] and Kurshin [17]. Later we shall use the paper by Benthem [13], the essential results of which we shall briefly reproduce.

We shall consider the case of plane strain. Transition to a state of plane stress can be effected by the usual replacement of Poisson's ratio σ by $\sigma/(1+\sigma)$. We calculate the singularity not by using the solutions of the corresponding problem for a wedge, but by starting directly from the boundary-value problem only. In this we shall use a system of polar co-ordinates σ and θ. Since near the corner $\sigma_\theta = 0$, $\tau_{\rho\theta} = 0$ when $\sigma = 0$, we have

$$\frac{\partial^2\Phi}{\partial\rho^2}=0, \qquad \frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\Phi}{\partial\rho}\right)=0, \qquad \theta=0 \tag{2.1}$$

Here Φ is a stress function satisfying the equation

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{1}{\partial\theta^2}\right)\left(\frac{\partial^2\Phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\Phi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\theta^2}\right)=0 \tag{2.2}$$

The boundary conditions along the clamped edge are of the form (the displacements being zero)

$$\frac{1}{\rho}\frac{\partial\Phi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\theta^2}-\frac{\sigma}{1-\sigma}\frac{\partial^2\Phi}{\partial\rho^2}=0 \tag{2.3}$$

$$\frac{\partial^2}{\partial\rho^2}\left(\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}\right)+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial\Phi}{\partial\theta}\right)+\frac{1-\sigma}{2\rho}\left(\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\rho\,\partial\theta}+\frac{1}{\rho^2}\frac{\partial^3\Phi}{\partial\theta^3}-\frac{\sigma}{1-\sigma}\frac{\partial^3\Phi}{\partial\rho^2\,\partial\theta}\right)=0$$

Separating the variables in (2.2), we obtain

$$\Phi=\rho^{s+1}F(s,\theta), \qquad F(s,\theta)=A_1\sin(s+1)\theta+$$
$$+A_2\cos(s+1)\theta+A_3\sin(s-1)\theta+A_4\cos(s-1)\theta$$

Here A_1, A_2, A_3, and A_4 are arbitrary constants.

By satisfying the four boundary conditions (2.1), (2.2) and (2.3) we obtain a restriction imposed upon s, namely

$$2\varkappa\cos\pi s-4s^2+1+\varkappa^2=0, \quad \varkappa=3-4\sigma \tag{2.4}$$

For any values of σ within $0<\sigma<0.5$, equation (2.4) has a positive root
For $\sigma=0{\cdot}31741$, $s_0=0{\cdot}70000$, and the stress in the corner when $\rho\to0$ increases like $\rho^{-0.3}\psi(\theta)$, where $\psi(\theta)$ is a bounded function.

3. As a numerical method of solving the integral equation (1.13) we shall use an analogue of the method of Mul'topp-Kalandiia [18 and 19]. This method was developed in connection with the approximate solution of the equation of the theory of a wing of finite span. We shall seek an approximate solution to the problem in the form

$$\sigma(y)=(1-y^2)^{s_0-1}\sum_{k=1}^{N+1}A_k y^{2(k-1)} \tag{3.1}$$
$$(s_0=0.70000 \text{ for } \sigma=0.31741)$$

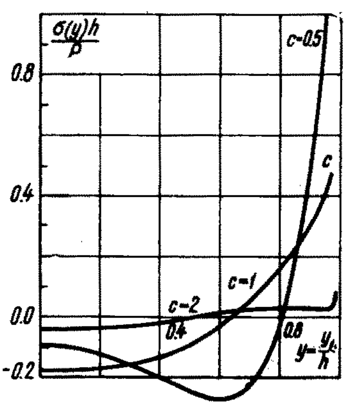

FIG. 2.

Substituting Expression (3.1) in the integral equation (1.13) and equating the left- and right-hand sides at the nodal points y_k selected in a definite manner, we obtain a system of linear algebraic equations in a_k. As the nodes of collocation we selected the roots of the Chebyshev polynomials

$$y_k = \cos \frac{2k-1}{4N} \pi \qquad (k = 1, \ldots, N) \tag{3.2}$$

Here N is the number of subdivision points in the interval $[0, 1]$. In this way the problem has been solved for the case of compression of a semi-infinite strip by two concentrated forces P $(g_1 = (P / G) \sin \lambda c,$ $r_1 = 0,$ $C_3 = C_0 (P / h),$ with the supplementary condition

$$\int_{-1}^{1} \sigma(t)\, dt = 0 \tag{3.3}$$

The integral expressions in (1.13) which contain singular kernels were evaluated by expressing them in terms of hypergeometric series [20], the other expressions being evaluated by Simpson's rule. As a result a linear system of algebraic equations of the following form was obtained:

$$\sum_{i=1}^{N+1} a_{ik} A_i = B_k \qquad (k = 1, \ldots, N+1) \tag{3.4}$$

Here N is the number of subdivision points in the interval $[0, 1]$ and B_k are absolute terms; the values of the elements of the matrix of coefficients $\| a_{ik} \|$ are as follows:-

For $N = 7$

3.79797	0.679865	0.133778	−0.069639	−0.166754	−0.219020	−0.248936	1.000000
3.84433	0.845496	0.340080	0.159890	0.077913	0.036060	0.013403	1.000000
3.93114	1.133346	0.675770	0.508745	0.425283	0.374819	0.340033	1.000000
4.04211	1.493278	1.045873	0.849866	0.730660	0.646622	0.582733	1.000000
4.15334	1.860771	1.359040	1.102649	0.935391	0.818815	0.731779	1.000000
4.24467	2.162253	1.564801	1.250573	1.054180	0.918675	0.818839	1.000000
4.29476	2.332245	1.660292	1.319084	1.109272	0.965474	0.859885	1.000000
1.00000	0.416667	0.284091	0.221946	0.184955	0.160057	0.141986	0.000000

For $N = 6$

3.800071	0.687903	0.144033	−0.0579843	−0.154076	−0.205535	1.000000
3.861710	0,907760	0.415292	0.240943	0.161525	0.120365	1.000000
3.975676	1.277911	0.831596	0.658257	0.563429	0.500690	1.000000
4.108963	1.711400	1.239934	1.009266	0.862153	0.757591	1.000000
4.225268	2.096990	1.524200	1.221575	1.031080	0.899142	1.000000
4.292418	2.324274	1.656142	1.316071	1.106832	0.963398	1.000000
1.000000	0.416667	0.284091	0.221946	0.184955	0.160057	0.000000

For $N = 5$

3.803561	0.701064	0.160750	−0.039054	−0.133544	1.000000
3.892630	1.006447	0.531682	0.363203	0.284434	1.000000
4.042105	1.493278	1.045873	0.849866	0.730660	1.000000
4.194284	1.993357	1.455256	1.171876	0.991675	1.000000
4.288577	2.311127	1.649231	1.311061	1.102781	1.000000
1.000000	0.416667	0.284091	0.221946	0.184955	0.000000

for $C = \frac{C_1}{h} = 0.5$, $\sigma = 0.31741$

Equation (3.4) corresponding to the number $N + 1$ is an expression of condition (3.3), and $A_{N+1} = C_0$, $B_{N+1} = 0$.

Table 1 gives the results of the calculations for the stress $\sigma(y)$ at the clamp for various values of the number of collocation points ($N = 5, 6, 7$) for $C = 0{\cdot}5$ and $\sigma = 0{\cdot}31741$.

Table 1

y	$\sigma(y)h/P$		
	$N=5$	$N=6$	$N=7$
0	−0.09355	−0.09226	−0.09260
0.1	−0.10045	−0.09938	−0.09962
0.2	−0.12145	−0.12037	−0.12048
0.4	−0.20207	−0.19847	−0.19861
0.6	−0.26920	−0.27203	−0.27055
0.8	−0.03490	−0.00570	−0.00820
0.95	+0.97350	+0.98192	+0.98662

Table 2

A_k	$C=0.5$	$C=1$	$C=2$
A_1	−0.09260	−0.17953	−0.03621
A_2	−0.67577	+0.24741	+0.17426
A_3	+0.36628	+0.64034	−0.11729
A_4	+0.31330	+0.04506	−0.01961
A_5	+2.47072	−1.74094	+0.02093
A_6	−0.11710	+2.21550	−0.00727
A_7	−1.67375	−1.04300	−0.01006
C_0	+0.64518	+0.77436	+0.79367

It can be seen from these tables that the fifth approximation differs from the sixth by 2% and the sixth from the seventh by 0·8%. It was found that the smaller the value of $C = C_1 / h$, the larger is the number of terms which must be taken in (3.1) to attain the same accuracy.

Table 2 contains values of the coefficients A_k for the seventh approximation and for values of $C = 0{\cdot}5$, 1·0 and 2·0.

Figure 2 shows graphs of the variation of the stress at the clamp against $C = C_1 / h$.

It can be seen that as $C \to 0$ the stresses tend to zero non-uniformly over the thickness of the strip and in such a way that they become more and more concentrated in the corner.

In practice St. Venant's principle is already applicable for all $C = C_1 / h \geqslant 2$, i.e. beyond this limit the conditions of clamping have an insignificant effect on the solution.

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